Single Neuron Dynamics —
Models Linking Theory and Experiment

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Conductance-Based Models

In the following the specifications of the conductance-based models used in this thesis are given. All potentials are measured in mV, conductances in mS/cm$^2$, capacitances in $\mu$F/cm$^2$, and currents in $\mu$A/cm$^2$.

### A–1 Hodgkin-Huxley model

The original model of Hodgkin & Huxley (1952) with the resting potential set to $-65$ mV. The Hodgkin-Huxley model is an example of a class-II neuron.

$$C\dot{V} = -I_{Na} - I_{K} - I_{L} + I$$

Membrane capacitance: $C = 1$ $\mu$F/cm$^2$.

**Sodium current**

$$I_{Na} = \tilde{g}_{Na} m^3 h (V - E_{Na})$$ $$m = \alpha_m(V)(1 - m) - \beta_m(V)m$$ $$h = \alpha_h(V)(1 - h) - \beta_h(V)h$$

$\tilde{g}_{Na} = 120$ mS/cm$^2$, $E_{Na} = +50$ mV, 
$\alpha_m(V) = 0.1(V + 40)/(1 - \exp(-(V + 40)/10)),$ 
$\beta_m(V) = 4\exp(-(V + 65)/18),$ 
$\alpha_h(V) = 0.07\exp(-(V + 65)/20),$ 
$\beta_h(V) = 1/(1 + \exp(-(V + 35)/10)).$

**Potassium delayed-rectifier current**

$$I_{K} = \tilde{g}_{K} n^4 (V - E_{K})$$ $$\dot{n} = \alpha_n(V)(1 - n) - \beta_n(V)n$$

$\tilde{g}_{K} = 36$ mS/cm$^2$, $E_{K} = -77$ mV, 
$\alpha_n(V) = 0.01(V + 55)/(1 - \exp(-(V + 55)/10)),$ 
$\beta_n(V) = 0.125\exp(-(V + 65)/80).$
Leakage current

\[ I_L = \bar{g}_L(V - E_L) \]

\( \bar{g}_L = 0.3 \text{ mS/cm}^2, E_L = -54.384 \text{ mV} \).

A–2 Traub-Miles model

A version of the model of Traub et al. (1991) is used in chapter 3, which has only a single compartment and includes only the sodium, delayed rectifier and leakage current. This model is a simple example of a class-I neuron. Note that it contains the same currents as the Hodgkin-Huxley model. Only their parameters are slightly changed. The resting potential is at \( V = -66.6 \text{ mV} \).

\[ C\dot{V} = -I_{Na} - I_K - I_L + I \]

Membrane capacitance: \( C = 1 \text{ μF/cm}^2 \)

Sodium current

\[
\begin{align*}
I_{Na} & = \bar{g}_{Na}m^3h(V - E_{Na}) \\
n & = \alpha_m(V)(1 - m) - \beta_m(V)m \\
h & = \alpha_h(V)(1 - h) - \beta_h(V)h
\end{align*}
\]

\( \bar{g}_{Na} = 100 \text{ mS/cm}^2, E_{Na} = +48 \text{ mV} \),
\( \alpha_m(V) = 0.32(V + 54)/(1 - \exp(-(V + 54)/4)), \)
\( \beta_m(V) = 0.28(V + 27)/(\exp((V + 27)/4) - 1), \)
\( \alpha_h(V) = 0.128 \exp(-(V + 50)/18), \)
\( \beta_h(V) = 4/(1 + \exp(-(V + 27)/5)). \)

Potassium delayed-rectifier current

\[
\begin{align*}
I_K & = \bar{g}_K n^4(V - E_K) \\
n & = \alpha_n(V)(1 - n) - \beta_n(V)n
\end{align*}
\]

\( \bar{g}_K = 200 \text{ mS/cm}^2, E_K = -82 \text{ mV} \),
\( \alpha_n(V) = 0.032(V + 52)/(1 - \exp(-(V + 52)/5)), \)
\( \beta_n(V) = 0.5 \exp(-(V + 57)/40). \)

Leakage current

\[ I_L = \bar{g}_L(V - E_L) \]

\( \bar{g}_L = 0.1 \text{ mS/cm}^2, E_L = -67 \text{ mV} \).
A–3  Modified Traub-Miles model

A variant of the Traub-Miles model from above is used in chapter 4.

\[ CV = -I_{Na} - I_{K} - I_{L} - I_{Ca} + I \]

There are only slight differences in the conductances and reversal potentials of the sodium and the potassium current (Ermentrout, 1998) and an additional calcium current.

\[ \bar{g}_{Na} = 100 \text{mS/cm}^2, \quad E_{Na} = +50 \text{mV}, \]
\[ \bar{g}_{K} = 80 \text{mS/cm}^2, \quad E_{K} = -100 \text{mV}, \]
\[ \bar{g}_{L} = 0.1 \text{mS/cm}^2, \quad E_{L} = -67 \text{mV}. \]

**Calcium current**

\[ I_{Ca} = \bar{g}_{Ca} s(V - E_{Ca}) \]
\[ s = \frac{1}{1 + \exp(-(V + 25)/5))} \]

\[ \bar{g}_{Ca} = 5 \text{mS/cm}^2, \quad E_{Ca} = 120 \text{mV}. \]

**Traub-Miles model with M-type current**

An M-type current was added to the modified Traub-Miles model to simulate spike-frequency adaptation (Ermentrout, 1998):

\[ CV = -I_{Na} - I_{K} - I_{L} - I_{Ca} - I_{M} + I \]

**M-type current**

\[ I_{M} = \bar{g}_{M} w(V - E_{M}) \]
\[ \tau_{w}(V) \dot{w} = w_{\infty}(V) - w \]

\[ \bar{g}_{M} = 8 \text{mS/cm}^2, \quad E_{M} = -100 \text{mV}, \]
\[ \tau_{w}(V) = 100 \text{ms}, \]
\[ w_{\infty}(V) = \frac{1}{1 + \exp(-(V + 20)/5)}. \]

**Traub-Miles model with AHP current**

Alternatively, an AHP current and calcium dynamics was added to the modified Traub-Miles model (Ermentrout, 1998):

\[ CV = -I_{Na} - I_{K} - I_{L} - I_{Ca} - I_{AHP} + I \]

**AHP current and calcium dynamics**

\[ I_{AHP} = \bar{g}_{AHP} q(V - E_{AHP}) \]
\[ q = [Ca]/(30 + [Ca]) \]
\[ [Ca] = -0.002I_{Ca} - 0.0125[Ca] \]

\[ \bar{g}_{AHP} = 4 \text{mS/cm}^2, \quad E_{AHP} = -100 \text{mV}. \]
A–4 Connor model

The A-current is a potassium current, which is present in many neurons. Connor et al. (1977) added this current to the Hodgkin-Huxley model. The resulting equations show class-I neuron properties. The resting potential of the Connor model is at \( V = -73 \) mV.

\[
CV = -I_{Na} - I_{K} - I_{A} - I_{L} + I
\]

Membrane capacitance: \( C = 1 \) \( \mu \)F/cm\(^2\).

**Sodium current**

\[
I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na})
\]

\[
m = \alpha_m(V) (1 - m) - \beta_m(V) m
\]

\[
h = \alpha_h(V) (1 - h) - \beta_h(V) h
\]

\( \bar{g}_{Na} = 120 \) mS/cm\(^2\), \( E_{Na} = +50 \) mV,

\( \alpha_m = 0.1(V + 34.7)/(1 - \exp(-(V + 34.7)/10)) \),

\( \beta_m = 4 \exp(-(V + 59.7)/18) \),

\( \alpha_h = 0.07 \exp(-(V + 53)/20) \),

\( \beta_h = 1/(1 + \exp(-(V + 23)/10)) \).

**Potassium delayed-rectifier current**

\[
I_{K} = \bar{g}_K n^4 (V - E_{K})
\]

\[
n = \alpha_n(V) (1 - n) - \beta_n(V) n
\]

\( \bar{g}_K = 20 \) mS/cm\(^2\), \( E_{K} = -77 \) mV,

\( \alpha_n(V) = 0.005(V + 50.7)/(1 - \exp(-(V + 50.7)/10)) \),

\( \beta_m(V) = 0.0625 \exp(-(V + 60.7)/80) \).

**Potassium A-current**

\[
I_{A} = \bar{g}_{A} a^3 b (V - E_{A})
\]

\[
\tau_a(V) \dot{a} = a_w(V) - a
\]

\[
\tau_b(V) \dot{b} = b_w(V) - b
\]

\( \bar{g}_{A} = 47.7 \) mS/cm\(^2\), \( E = -80 \) mV,

\( a_w(V) = (0.0761 \exp((V + 99.22)/31.84)/(1 + \exp((V + 6.17)/28.93)))^{1/3} \),

\( \tau_a(V) = 0.3632 + 1.158/(1 + \exp((V + 60.96)/20.12)) \),

\( b_w(V) = 1/(1 + \exp((V + 58.3)/14.54))^2 \),

\( \tau_b(V) = 1.24 + 2.678/(1 + \exp((V + 55)/16.072)) \).

**Leakage current**

\[
I_{L} = \bar{g}_L (V - E_{L})
\]

\( \bar{g}_L = 0.3 \) mS/cm\(^2\), \( E_{L} = -22 \) mV.
A–5  Crook model

The model of Crook et al. (1998) was used as an additional example of an adapting neuron. It is a two-compartment model. One compartment is the membrane equation for the potential $V_s$ of the soma. It contains all the voltage dependent currents for the generation of spikes and possible adaptation currents. The other compartment is a linear membrane equation and models the whole dendritic tree. Its potential is $V_d$. Both compartments are coupled by the coupling current $I_C$. Note that the input current $I$ is injected into the soma. Therefore the adaptation currents are still additive to the input current. The resting potential of the Crook model is at $-77$ mV.

$$CV_s = -I_{Na} - I_{K} - I_{Ca} - I_{LS} - I_C/P + I/P$$
$$CV_d = -I_{LD} + I_C/(1 - P)$$

Membrane capacitance: $C = 0.8 \mu F/cm^2$. Proportion of the cell area taken up by the soma: $P = 0.05$.

**Sodium current**

$$I_{Na} = \bar{g}_{Na} m^2 h (V_s - E_{Na})$$
$$\dot{m} = \alpha_m(V)(1 - m) - \beta_m(V)m$$
$$h = \alpha_h(V)(1 - h) - \beta_h(V)h$$

$\bar{g}_{Na} = 221 \text{mS/cm}^2$, $E_{Na} = +55$ mV, 
$\alpha_m(V) = 0.32(-47.1 - V_s)/(\exp(0.25(-47.1 - V_s)) - 1)$, 
$\beta_m(V) = 0.28(V_s + 20.1)/(\exp((V_s + 20.1)/5) - 1)$, 
$\alpha_h(V) = 0.128 \exp((-43 - V_s)/18)$, 
$\beta_h(V) = 4/\exp((-20 - V_s)/5) + 1)$. 

**Potassium delayed-rectifier current**

$$I_{K} = \bar{g}_K n (V_s - E_K)$$
$$\dot{n} = \alpha_n(V)(1 - n) - \beta_n(V)n$$

$\bar{g}_K = 47 \text{mS/cm}^2$, $E_K = -90$ mV, 
$\alpha_n(V) = 0.59(-25.1 - V_s)/(\exp((-25.1 - V_s)/5) - 1)$, 
$\beta_n(V) = 0.925 \exp(0.925 - 0.025(V_s + 77))$. 

**Calcium current**

$$I_{Ca} = \bar{g}_{Ca} s^2 r (V_s - E_{Ca})$$
$$\dot{s} = \alpha_s(V)(1 - s) - \beta_s(V)s$$
$$\tau_r(V) \dot{r} = r_\infty(V) - r$$

$\bar{g}_{Ca} = 8.5 \text{mS/cm}^2$, $E_{Ca} = +120$ mV, 
$\alpha_s(V) = 0.912/\exp(-0.072(V_s - 5)) + 1)$, 
$\beta_s(V) = 0.0114(V_s + 8.9)/(\exp((V_s + 8.9)/5) - 1)$, 
$r_\infty(V) = \min(\exp(-(V_s + 60)/20), 1)$, 
$\tau_r(V) = 200$ ms.
**Soma leakage-current**
\[ I_{LS} = \tilde{g}_{LS}(V_s - E_{LS}) \]
\[ \tilde{g}_{LS} = 2 \text{ mS/cm}^2, E_{LS} = -70 \text{ mV}. \]

**Dendrite leakage-current**
\[ I_{LD} = \tilde{g}_{LD}(V_d - E_{LD}) \]
\[ \tilde{g}_{LD} = 0.05 \text{ mS/cm}^2, E_{LD} = -70 \text{ mV}. \]

**Coupling current**
\[ I_C = \tilde{g}_C(V_s - V_d) \]
\[ \tilde{g}_C = 1.1 \text{ mS/cm}^2 \]

**Crook model with M-type current**
To simulate the effect of an M-type adaptation current it was added to the Crook-model.
\[ CV_s = -I_{Na} - I_K - I_{Ca} - I_M - I_{LS} - I_C/P + 1/P \]

**M-type current**
\[ I_M = \tilde{g}_M w(V_s - E_K) \]
\[ \tau_w(V) = \frac{w_\infty(V) - w}{\exp(V_s - 35) + 1} \]
\[ \tilde{g}_M = 6.5 \text{ mS/cm}^2, E_K = -90 \text{ mV}, \]
\[ w_\infty(V) = \frac{1}{\exp(-(V_s + 35)/10) + 1}, \]
\[ \tau_w(V) = 92 \exp(-(V_s + 35)/20) / (1 + 0.3 \exp(-(V_s + 35)/10)). \]

**Crook model with AHP current**
To simulate the effect of an AHP-type adaptation current it was added together with the calcium dynamics to the Crook-model.
\[ CV_s = -I_{Na} - I_K - I_{Ca} - I_{AHP} - I_{LS} - I_C/P + 1/P \]

**AHP current and calcium dynamics**
\[ I_{AHP} = \tilde{g}_{AHP} q(V_s - E_K) \]
\[ \tau_q(V) = \frac{q_\infty(V) - q}{\exp(V_s - 35) + 1} \]
\[ [Ca] = -B I_{Ca} - [Ca]/\tau_{Ca} \]
\[ \tilde{g}_{AHP} = 7 \text{ mS/cm}^2, E_K = -90 \text{ mV}, \]
\[ q_\infty(V) = (0.0005[Ca])^2, \]
\[ \tau_q(V) = 0.0338 / (\min(0.00001[Ca], 0.01) + 0.001), \]
\[ B = 3, \tau_{Ca} = 60 \text{ ms}. \]